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MOORE SCHOOL OF ELECTRICAL ENGINEERING PHILADELPHIA P--ETC F/6 17/1
PHASE VARIATION WITH POSITION IN AN UNDERWATER MULTIPATH ENVIRO--ETC(U)
FEB 78 F HABER N00014-77-C-0252

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Fred/Haber, Phase Variation with Position in an Underwater Multipath Environment and its Effect on Array Pattern, Valley Forge Research Center, QPR No. 24, February 1978.

QPR 24

LEVEL 1

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AD A100742

-- 11 - TITLE (U) RESEARCH IN DISTRIBUTED ARRAYS
-- 1 - AGENCY ACCESSION NO. DN775290
-- 2 - DATE OF SUMMARY 12 NOV 80
-- 39 - PROCESSING DATE (RANGE) 30 NOV 80
-- 6 - SECURITY OF WORK UNCLASSIFIED
-- 12 - S + T AREAS
-- 000200 ACOUSTICS
-- 000100 ACOUSTIC DETECTION
-- 21E - MILITARY/CIVILIAN APPLICATIONS MILITARY
-- 10A1 - PRIMARY PROGRAM ELEMENT 62711N
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-- 10A2A - PRIMARY PROJECT AGENCY AND PROGRAM F11121
-- 10A3 - PRIMARY TASK AREA RF11121810
-- 10A4 - WORK UNIT NUMBER NR-129-102
-- 17A1 - CONTRACT/GRANT EFFECTIVE DATE MAR 77
-- 17A2 - CONTRACT/GRANT EXPIRATION DATE FEB 80
-- 17B - CONTRACT/GRANT NUMBER N00014-77-C-0252
-- 17C - CONTRACT TYPE COST TYPE
-- 17D2 - CONTRACT/GRANT AMOUNT \$ 25,000
-- 17E - KIND OF AWARD EXT
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As a preliminary we examine a two-ray condition as shown in Figure 4.* One wavefront is assumed to arrive along the x-axis, another at an angle θ . The wavefronts are sinusoidal in time and the sum at points along x is

$$s(t,x) = A_1 \cos(\omega t - kx) + A_2 \cos(\omega t - \phi - kx \cos \theta) \quad (1)$$

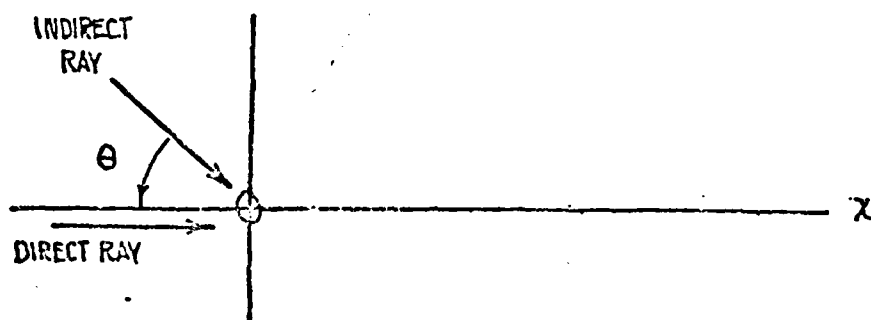


FIGURE 4. RAY ARRIVALS

* The case treated here is analogous to one encountered in FM systems with sinusoidal interference to a desired carrier.

A_1 and A_2 represent the magnitude of the two wavefronts, ϕ is the phase difference between them at $x = 0$, and $k = 2\pi/\lambda$ where λ is the wavelength. A sensor at point x responding equally and linearly to both wavefronts would see $s(t, x)$. Writing (1) in envelope-angle form,

$$s(t, x) = B(x) \cos(\omega t + \psi(x)) \quad (2)$$

where

$$B(x) = A_1^2 + A_2^2 + 2A_1A_2 \cos[\phi - kx(1 - \cos\theta)] \quad (3)$$

$$\psi(x) = \tan^{-1} \frac{A_1 \sin kx + A_2 \sin(\phi + kx \cos\theta)}{A_1 \cos kx + A_2 \cos(\phi + kx \cos\theta)} \quad (4)$$

The phase obtained using (4) will be modulo- 2π . It is useful to deal with the phase derivative $d\psi/dx$ if the modulo- 2π ambiguity is to be avoided. It can be shown that

$$\frac{d\psi}{dx} = k - \frac{k}{2}(1 - \cos\theta) \left[1 + \frac{1 - a^2}{1 + a^2 + 2a \cos[\phi - kx(1 - \cos\theta)]} \right] \quad (5)$$

where

$$a = A_1/A_2 \quad (6)$$

When a is large, meaning that the important part of the received wave is along the x -axis

$$\frac{d\psi}{dx} = k - \frac{k(1 - \cos\theta)}{a} \cos[\phi - kx(1 - \cos\theta)], \quad (7)$$

It fluctuates sinusoidally around k with the fluctuation amplitude decreasing to zero as a goes to infinity. For a small

$$\frac{d\psi}{dx} = k \cos\theta - ka(1 - \cos\theta) \cos[\phi - kx(1 - \cos\theta)], \quad (8)$$

again a sinusoidal fluctuation which decreases to zero as a approaches zero. For intermediate values of a the fluctuation of $d\psi/dx$ is as shown in Figure 5.

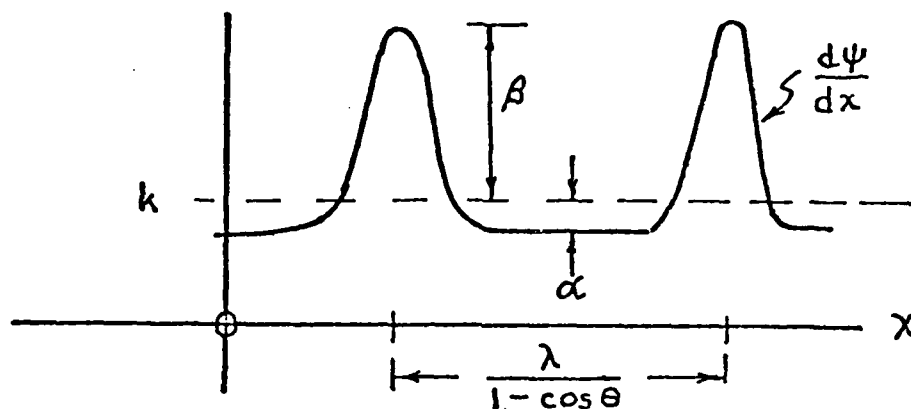


FIGURE 5. PHASE DERIVATIVE VARIATION WITH DISTANCE IN THE TWO-RAY CASE.

The variation is periodic with period

$$X = \frac{\lambda}{1 - \cos \theta} \quad (9)$$

and has peak excursions above and below k given by

$$\beta = k(1 - \cos \theta)/(a - 1) \quad (10)$$

$$\alpha = -k(1 - \cos \theta)/(a + 1) \quad (11)$$

Figure 5 is drawn assuming $a > 1$. The fluctuation is around k , which turns out to be the average of $d\psi/dx$. Note that at $a = 1^+$ β is positive and high in magnitude, and the fluctuation is highly impulsive. For $a < 1$, $d\psi/dx$ fluctuates around the value $k(1 - \cos \theta)$ rather than around k and for $a = 1^-$ β is negative and high in magnitude, and the fluctuation is again impulsive but negative going. When a is close to unity the phase as a function of position (which is the integral of $d\psi/dx$) is as shown in Figure 6.

In underwater applications the angle θ typically found in long range paths is less than 20° . Assuming it to be 10° the period is

$$X = \frac{\lambda}{1 - 0.98} = 50\lambda \quad (12)$$

It is interesting to note that in analyses found in the literature of the correlation distance of underwater acoustic waves, numerical estimates around 50λ are typically obtained (see for instance [1]).

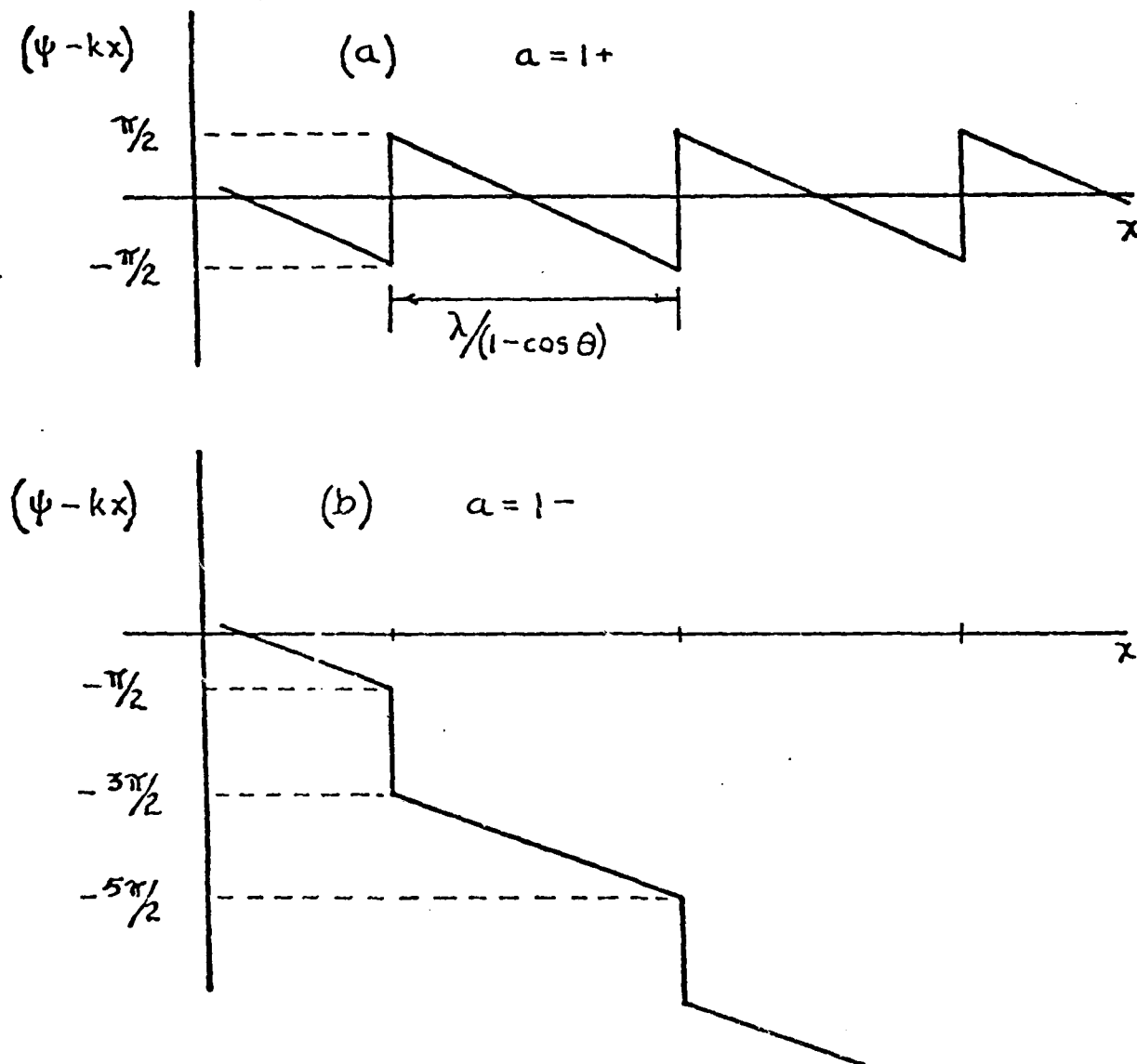


FIGURE 6. PHASE AS A FUNCTION OF POSITION FOR NEARLY EQUAL MAGNITUDE RAYS.

- [1] P. W. Smith, Jr., "Spatial Coherence in Multipath or Multimodel Channels," *Journal Acoustic Society of America*, Vol. 60, No. 2, Aug. 1976, pp. 305-310.

We now turn to the more general case of n wavefronts arriving at angles θ_j , $j = 1, 2, \dots, n$. The received sum at a position x is

$$s(t, x) = \sum_{j=1}^n A_j \cos(\omega t - \phi_j - kx \cos \theta_j) \quad (13)$$

The ϕ_j , $j = 1, 2, \dots, n$ are random phase angles of each of the wavefronts on arrival at the point $x = 0$. It is convenient to write this in the form

$$\begin{aligned} s(t, x) &= B(x) \cos[\omega t + \psi(x)] \\ &= \operatorname{Re} z(x) e^{j\omega t} \end{aligned} \quad (14)$$

where

$$z(x) = B(x) e^{j\psi(x)} = \sum_{j=1}^n A_j e^{j(\phi_j + kx \cos \theta_j)} \quad (15)$$

$\psi(x)$ is the phase angle we will study and as was done before we find the phase derivative

$$\frac{d\psi}{dx} = \operatorname{Im} \left(\frac{1}{z(x)} \frac{dz}{dx} \right) \quad (16)$$

From (15) we have

$$\frac{dz}{dx} = j \sum_{j=1}^n A_j k \cos \theta_j e^{j(\phi_j + kx \cos \theta_j)} \quad (17)$$

so that (16) becomes

$$\begin{aligned} \frac{d\psi}{dx} &= \operatorname{Im} j \frac{\sum_j A_j k \cos \theta_j e^{j(\phi_j + kx \cos \theta_j)}}{\sum_j A_j e^{j(\phi_j + kx \cos \theta_j)}} \\ &= \frac{\sum_i \sum_j A_i A_j k \cos \theta_i e^{j[\phi_i - \phi_j + kx(\cos \theta_i - \cos \theta_j)]}}{\left| \sum_j A_j e^{j(\phi_j + kx \cos \theta_j)} \right|^2} \\ &= \frac{\sum_i \sum_j A_i A_j k \cos \theta_i \cos[\phi_j - \phi_i + kx(\cos \theta_i - \cos \theta_j)]}{\sum_i \sum_j A_i A_j \cos[\phi_i - \phi_j + kx(\cos \theta_i - \cos \theta_j)]} \end{aligned} \quad (18)$$

As a rule $\theta_1 \ll \frac{\pi}{2}$ and $\cos\theta_1 = 1 - \frac{\theta_1^2}{2}$. Where this approximation is permissible we can write

$$\frac{d\psi}{dx} = k - \frac{k}{2} \frac{\sum_i \sum_j \theta_i^2 \alpha_{ij}(x)}{\sum_i \sum_j \alpha_{ij}(x)} \quad (19)$$

where

$$\alpha_{ij}(x) = A_i A_j \cos[\phi_i - \phi_j + kx(\cos\theta_i - \cos\theta_j)]$$

To retrieve ψ we must integrate $d\psi/dx$; i.e. we form

$$\int_0^x \frac{d\psi(x_1)}{dx_1} dx_1 = \psi(x) - \psi(0) \quad (20)$$

The integration will generate the phase difference between the phase at x and the phase at the origin of integration. From (19) we see that one term on integration will be kx , the linear phase variation associated with the normal phase vs. position function of a plane wave along the direction of travel of the wave. In beam forming with an array of sensors along x one will subtract the phase progression kx if the axis of the beam is to be colinear with the x axis. In this case the remaining phase difference between a point x and the origin is

$$-\frac{k}{2} \int_0^x \frac{\sum_i \sum_j \theta_i^2 \alpha_{ij}(x_1)}{\sum_i \sum_j \alpha_{ij}(x_1)} dx_1 \quad (21)$$

If the approximation $\cos\theta_i = 1 - \theta_i^2/2$ is not used, the remaining phase after correcting for kx is given by subtracting k from (18) and integrating over x .

Numerical evaluations of the remaining phase difference have been made for a number of cases.* One particular case is shown in Figure 7 determined assuming 21 equal amplitude rays arriving at 2° intervals from $\theta = -20^\circ$ to

*

Programming of this computation and the one described later giving array pattern, was done by De Juan Ho.

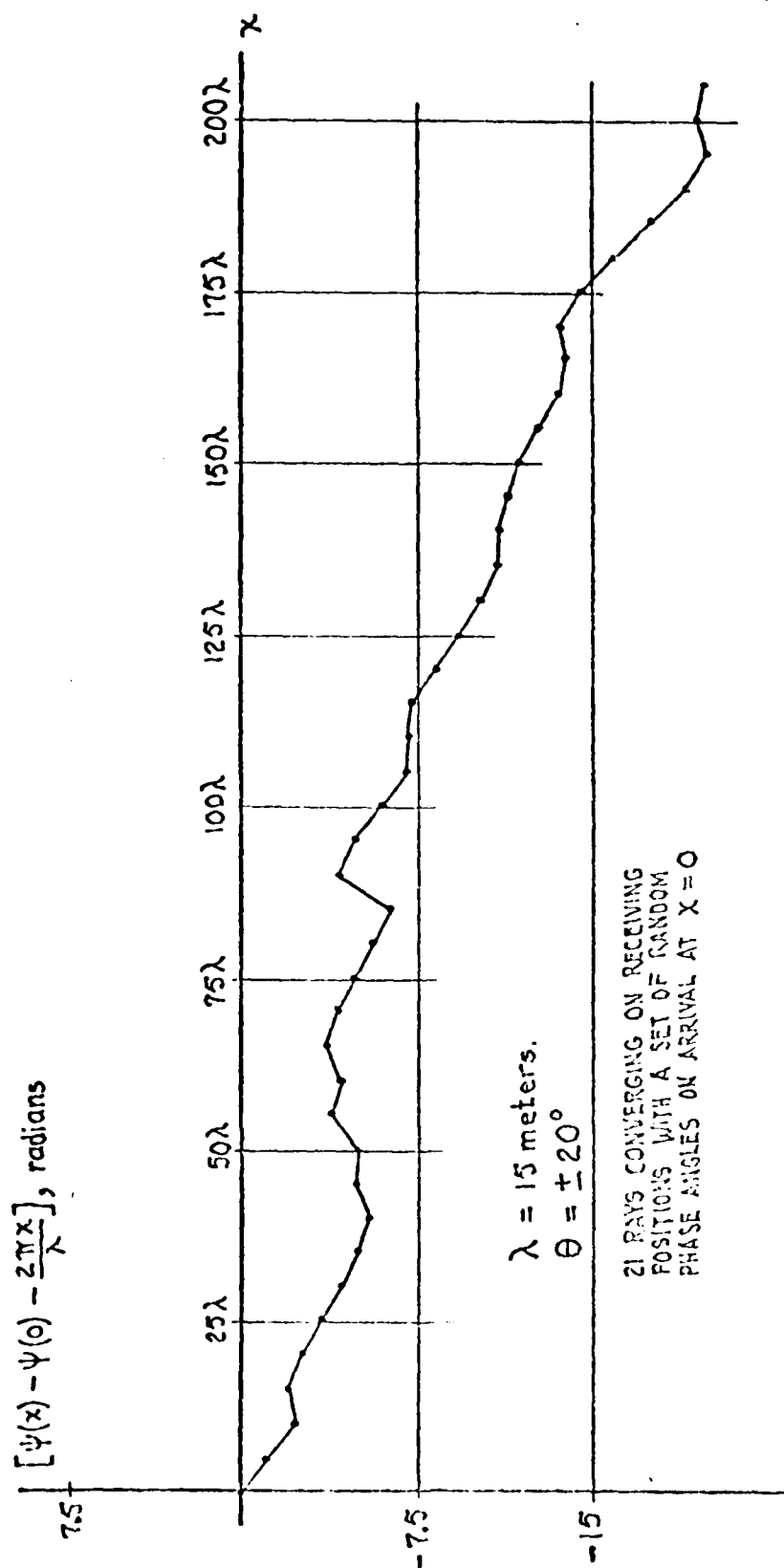


FIGURE 7. PHASE DIFFERENCE vs POSITION

$\pm 20^\circ$ with respect to the horizontal (see Figure 4), each with a phase angle ϕ randomly selected in the interval $(0, 2\pi)$. Because the average wavelength of the various rays as seen along x is less than λ , the wavelength along the direction of travel of the ray, there is a linearly-tending phase accumulation with distance as seen in Figure 7. On top of this accumulation there is a random variation. The fluctuation around a straight line approximation to the phase difference ranges around ± 3 radians. Thus even if the phase were corrected to account for the slope of the straight line approximation, a ± 3 radian random error would still be encountered.

Figure 7 was obtained with one randomly selected set of ray arrival phase angles. Additional examples will be ultimately computed for different sets of arrival phase angles to provide data suitable for obtaining statistical averages. Other cases, including different intervals of arrival angle, different ray amplitudes, and different numbers of arriving arrays will also be treated.

Having a sample function of phase vs. position, a logical next step is to determine the gain and pattern of the random planar floating array when it is focused in some azimuthal direction using conventional beamforming, and when the source signal is propagating toward the array through the multipath medium. As a first step a program was developed for selecting element positions over a circular area assuming a uniform distribution of element positions.

If the array is assumed confined to a circle of radius ρ with uniform distribution over the circle, the density function in the joint random variables \underline{X} , \underline{Y} , is

$$p_{X,Y}(x,y) = \frac{1}{\pi \rho^2} , \quad x^2 + y^2 \leq \rho^2$$

$$= 0 , \quad \text{elsewhere.}$$

Transforming to polar coordinates, (R, ϕ) , we have

$$p_{R,\phi}(r,\phi) = \frac{r}{\pi \rho^2} , \quad 0 \leq r \leq \rho$$

$$0 < \phi \leq 2\pi$$

$$= 0 , \quad \text{elsewhere}$$

The marginal densities in R and ϕ are

$$p_R(r) = \frac{2r}{\rho^2} \quad 0 \leq r \leq \rho$$

$$= 0 \quad , \quad \text{elsewhere}$$

$$p_\phi(\phi) = \frac{1}{2\pi} \quad , \quad 0 < \phi \leq 2\pi$$

$$= 0 \quad , \quad \text{elsewhere}$$

The random variables R and ϕ are independent and independent choices of these variables are made. Sample values of ϕ are obtained by a conventional computer program which selects sample values uniformly distributed in $(0,1)$ and multiplies these by 2π . Sample values of R are obtained by picking a number Z uniformly distributed in $(0,1)$ and forming

$$R = \rho Z^{1/2} ,$$

for then

$$p_R(r) = p_Z(z) \left| \frac{dz}{dr} \right| = \frac{2r}{\rho^2} , \quad 0 < r \leq \rho .$$

Finally, the pairs (r, ϕ) so obtained are converted back into rectangular coordinates by

$$x = r \cos \phi$$

$$y = r \sin \phi$$

Using element positions so determined the array pattern was next found. The geometry of the problem is shown in Figure 8.

Assuming N elements distributed over the circle, cophased to form a beam along the y axis, the array pattern is given by

$$A(\phi) = \frac{1}{N} \left| \sum_{n=1}^N B(d_n) e^{j[k(x_n \cos \phi + y_n \sin \phi - y_n) - \alpha(d_n)]} \right|$$

where x_n , y_n , ϕ , and d_n are defined in Figure 8, and $\alpha(d_n)$ is a phase vs position function of the form obtained earlier and shown in Figure 7. $B(d_n)$ is the amplitude of the acoustic field at the n th element. This quantity can be obtained using the earlier analysis but for our purposes now we will assume

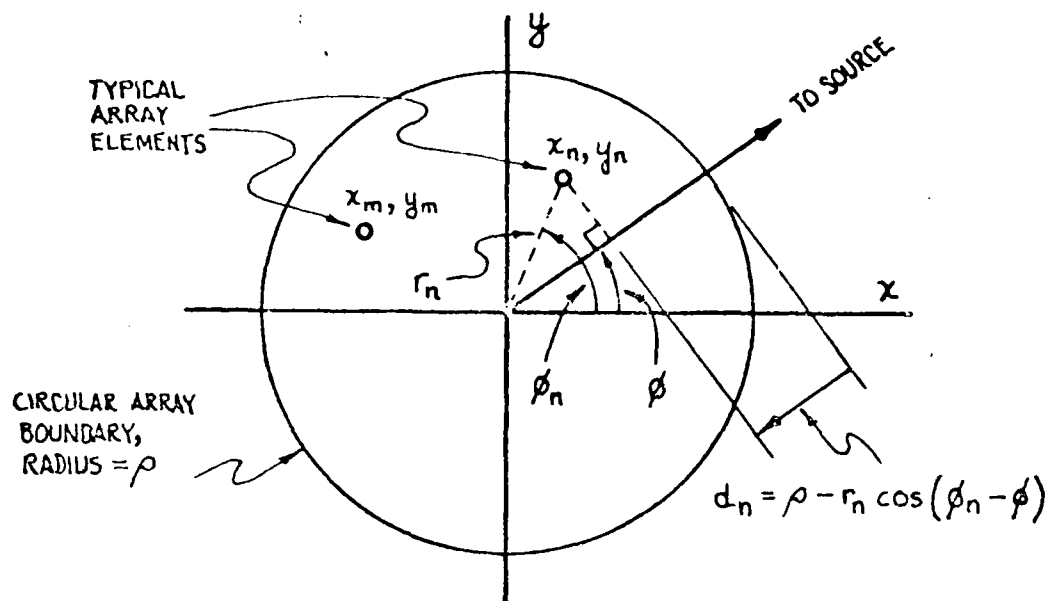


FIGURE 8. ARRAY GEOMETRY.

it constant and set it equal to unity for all n . Amplitude fluctuations as a rule, cause minor effects compared to phase fluctuations. The phase sample function of Figure 7, called now $\alpha(x)$ is used alone below to assess the effect of the multipath medium. The variable x in Figure 7 is replaced by d_n , with

$$\begin{aligned} d_n &= \rho - r_n \cos(\phi_n - \phi) \\ &= \rho - (x_n^2 + y_n^2)^{1/2} \cos(\tan^{-1} \frac{y_n}{x_n} - \phi) \end{aligned}$$

Computer calculations of $A(\phi)$, as described above, were carried out for two cases: (1) $\alpha(d_n) = 0$ and (2) $\alpha(d_n)$ as given by Figure 7, and the results are shown in Figures 9 and 10. Case 1 is that of propagation through a transparent (non-multipath) medium while case 2 is for the particular multipath case resulting in the phase function discussed above. Note that the gains along the main beam in the two cases are in the ratio of about 4.4 dB - a substantial factor; the sidelobe structure is different in detail but not in general characteristics. These results, it must be recognized, are based on one set of random arrival phases and on one set of random element positions; whether they are representative remains to be determined. Averaging over many sets

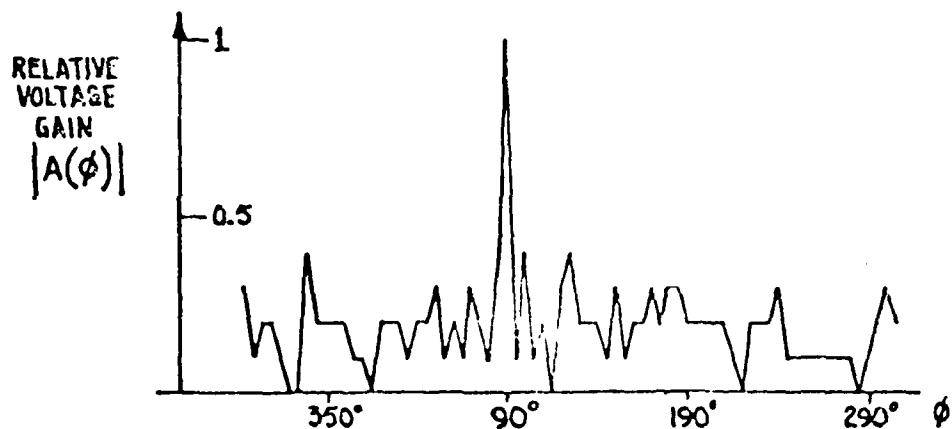


FIGURE 9. ARRAY PATTERN OF RANDOM PLANAR ARRAY IN TRANSPARENT MEDIUM. ENDFIRE BEAM FORMED AT AZIMUTH ANGLE, $\phi = 90^\circ$.

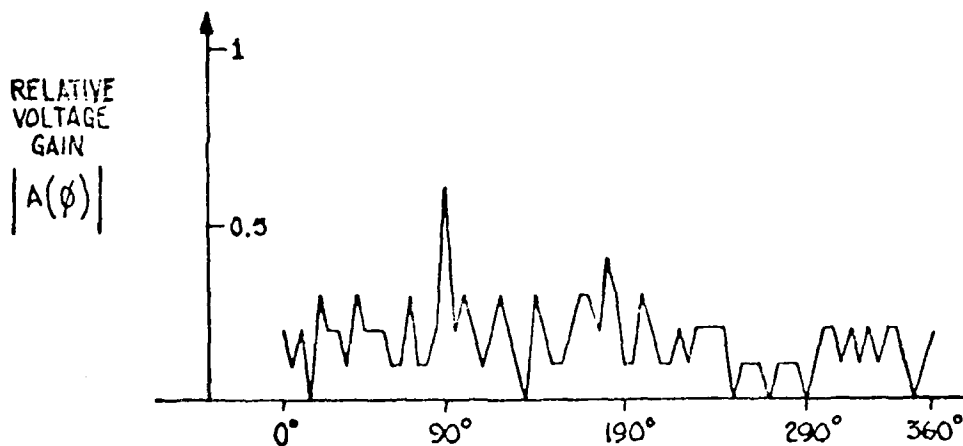


FIGURE 10. ARRAY PATTERN OF RANDOM PLANAR ARRAY ASSUMING MULTIPATH PROPAGATION. ENDFIRE BEAM FORMED AT AZIMUTH ANGLE $\phi = 90^\circ$ FOR TRANSPARENT MEDIUM.

of arrival phases and element positions, as well as carrying out additional computations with other system parameters, remain to be done.

Fred Haber

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